Task Specific Compressive Sensing for Target Detection

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Based on: Abhijit Mahalanobis and Mark Neifeld, "Optimizing measurements for feature-specific compressive sensing," Appl. Opt. 53, 6108-6118 (2014)

Basic Concept

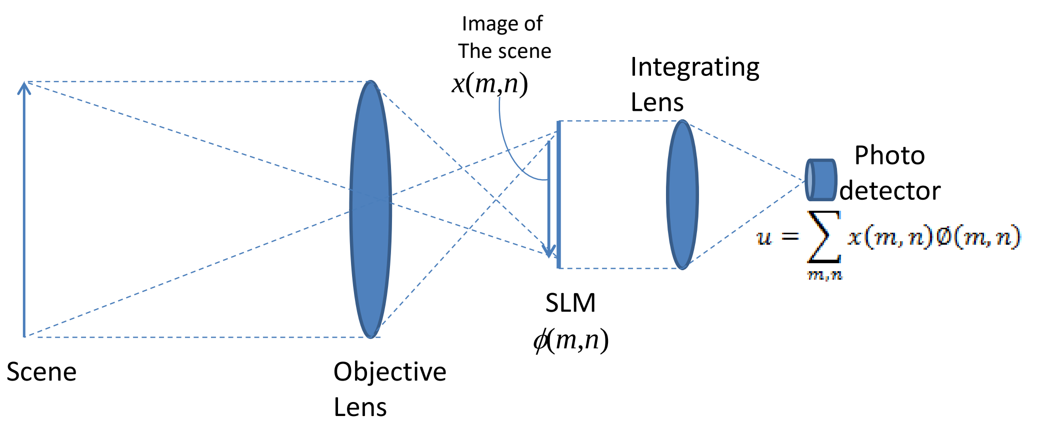

Figure 1: Basic architecture of a single pixel compressive sensor which integrates the light from the scene, modulated by codes written on the SLM

- Since real world SLMs can only implement non-negative masks, the values of $\emptyset(m,n)$ must range between 0 and 1
- To handle the negative values, the mask can be expressed in terms of two non-negative quantities, i.e,.

$$\emptyset(m,n) = \emptyset^{+}(m,n)\text{-}\emptyset^{-}(m,n),$$

where

$$\emptyset^{+}(m,n) = \begin{cases} \emptyset(m,n) \;\; if\; \emptyset(m,n) > 0 \\ 0\; otherwise \end{cases}$$

and

$$\emptyset^{-}(m,n) = \begin{cases} |\emptyset(m,n)| \;\; if\; \emptyset(m,n) < 0 \\ 0\; otherwise \end{cases}$$

What are the optimum masks for tasks specific compressive sensing ?

Linear re-reconstruction model

- Measurement model - $\boldsymbol{u} = \boldsymbol{\Phi x} + \boldsymbol{v}$
  - $\boldsymbol{R_v} = E\{\boldsymbol{vv^T}\}$, $\boldsymbol{R_x} = E\{\boldsymbol{xx^T}\}$
- The optimum linear reconstruction operator that minimizes $MSE = E\{|\boldsymbol{x} - \boldsymbol{Wu}|^2\}$ is given by $\boldsymbol{W} = \boldsymbol{R_x \Phi}^{\mathrm{T}}[\boldsymbol{\Phi R_x \Phi}^{\mathrm{T}} + \boldsymbol{R_v}]^{-1}$
  - $MSE = E\{|\boldsymbol{W}(\boldsymbol{\Phi x} + \boldsymbol{v}) - \boldsymbol{x}|^2\} = tr\,\{\boldsymbol{W\Phi R_x \Phi}^{\mathrm{T}}\boldsymbol{W}^T + \boldsymbol{WR_vW}^T + \boldsymbol{R_x}\} - 2tr\{\boldsymbol{\Phi R_x W}\}$
  - Optimum $\boldsymbol{W}$ (for any given choice of $\boldsymbol{\Phi}$) is obtained by setting the gradient of *MSE* to zero
- What is the optimum choice of $\boldsymbol{\Phi}$ which will yield the minimum value of *MSE for particular objects of interest*?

Single Measurement

- To answer this question, let us consider the special case where only one measurement is being made, and Φ is therefore a vector
  - $\boldsymbol{W} = \frac{\boldsymbol{R_x}\Phi^{\mathrm{T}}}{\alpha+\sigma}$
  - $MSE = tr\,\{\boldsymbol{R_x}\} - \frac{\Phi\boldsymbol{R_x}\boldsymbol{R_x}\Phi^{\mathrm{T}}}{\Phi\boldsymbol{R_x}\Phi^{\mathrm{T}}+\sigma}$
- **<u>Photon Constraint</u>**:
  - A consequence of the conservation of energy is that no column in the sensing matrix $\boldsymbol{\Phi}$ can sum (in absolute value) to greater than one

- $$\underset{over\ all\ \Phi}{minimize}\ \ MSE = tr\,\{\boldsymbol{R_x}\} - \frac{\Phi\boldsymbol{R_{xx}}\Phi^{\mathrm{T}}}{\Phi\boldsymbol{R_x}\Phi^{\mathrm{T}}+\sigma}$$

$$subject\ to:\ \max_j\left(\left|\Phi_j\right|\right) = 1$$

- $$\underset{over\ all\ \Phi}{maximize}\ J_{\sigma,\Phi} = \frac{\Phi\boldsymbol{R_{xx}}\Phi^{\mathrm{T}}}{\Phi\boldsymbol{R_x}\Phi^{\mathrm{T}}+\sigma}$$

$$subject\ to:\ \max_j\left(\left|\Phi_j\right|\right) = 1$$

Optimum solution for single measurement (cont'd)

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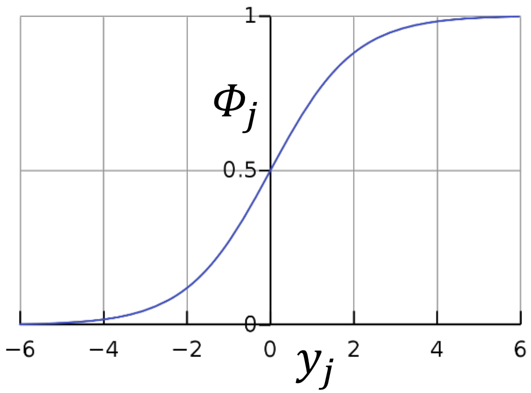
- One way to ensure that $|\Phi_j| \leq 1$ is to assume that $\Phi_j = \frac{1-exp(-\alpha y_j)}{1+exp(-\alpha y_j)}$, where y_j are *dummy variables* free to taken on any real value
  - we can maximize the ratio with respect to y_j, with the understanding that the mask values are a sigmoid function of the optimum values of y_j.



  - $J_\sigma = \frac{\Phi \boldsymbol{R_{xx}} \Phi^{\mathrm{T}}}{\Phi \boldsymbol{R_x} \Phi^{\mathrm{T}} + \sigma} = \frac{\sum_i \sum_j \Phi_i \Phi_j r_{ij}^{xx}}{\sum_i \sum_j \Phi_i \Phi_j r_{ij} + \sigma}$
  - $\boldsymbol{\nabla}_{y_i} J_\sigma = \frac{-2\alpha exp(-\alpha y_i)}{[1+exp(-\alpha y_i)]^2} \left[\frac{\sum_j \Phi_j r_{ij}^{xx}}{\sum_i \sum_j \Phi_i \Phi_j r_{ij} + \sigma} - \frac{\left[\sum_i \sum_j \Phi_i \Phi_j r_{ij}^{xx}\right]}{\left[\sum_i \sum_j \Phi_i \Phi_j r_{ij} + \sigma\right]^2} \sum_j \Phi_j r_{ij} \right]$
- $\boldsymbol{\nabla}_y J_\sigma = \frac{1}{\Phi \boldsymbol{R_x} \Phi^{\mathrm{T}} + \sigma} \cdot \mathbf{S} \cdot \boldsymbol{R_x} [\boldsymbol{R_x} \Phi^{\mathrm{T}} - J_\sigma \Phi^{\mathrm{T}}]$
  - Where $s_{ii} = \frac{-2\alpha exp(-\alpha y_i)}{[1+exp(-\alpha y_i)]^2}$

Observations

$$\nabla_y J_\sigma = \frac{1}{\Phi \boldsymbol{R_x} \Phi^{\mathrm{T}} + \sigma} \cdot \mathbf{S} \cdot \boldsymbol{R_x}[\boldsymbol{R_x}\Phi^{\mathrm{T}} - J_\sigma \Phi^{\mathrm{T}}]$$

- This gradient can be zero in one of two ways.
  - (i) $[\boldsymbol{R_x}\Phi^{\mathrm{T}} - J_\sigma \Phi^{\mathrm{T}}] = \mathbf{0}$**,** which implies that Φ^{T} is a eigen-vector of $\boldsymbol{R_x}$ with eigenvalue J_σ, or
  - (ii) the diagonal elements of $\mathbf{S}$ are zero.
- When σ is small, J_σ depends mostly on Φ.
  - In the limiting case when $\sigma = 0$, choosing Φ to be the dominant eigen-vector of $\boldsymbol{R_x}$ equates J_σ to its largest eigenvalue, and the first condition is satisfied.
- when σ becomes larger solution pivots to satisfy condition (ii),
  - $\frac{-2\alpha exp(-\alpha y_j)}{[1+exp(-\alpha y_j)]^2} = 0$, which implies that $\Phi_i = \frac{1-exp(-\alpha y_j)}{1+exp(-\alpha y_j)} = \pm 1$,
  - we have proof that the optimum values for the element of Φ must be binary
  - In this case, <u>use gradient descent </u>to find the optimum binary values of the mask

Additional Measurements

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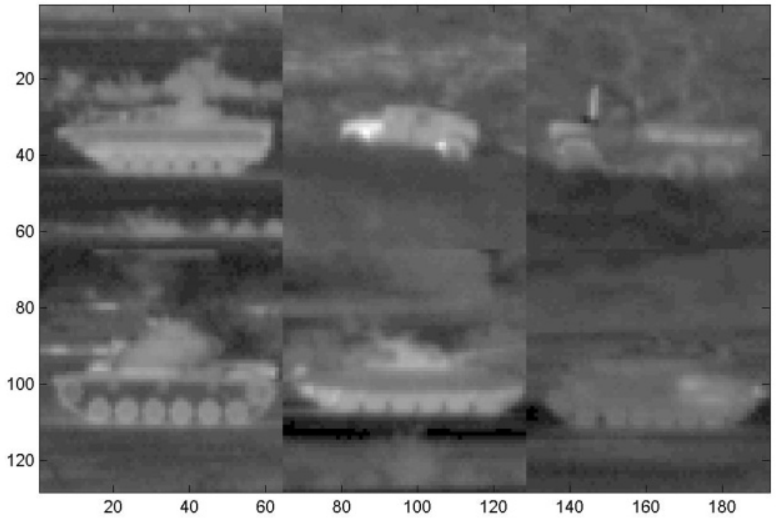
- To find the next best measurement vector, we "remove" the component of the signal that is represented by the first measurement, therefore
  - $\hat{\boldsymbol{x}} = \boldsymbol{x} - \frac{\boldsymbol{R_x}\Phi^{\mathrm{T}}\Phi\boldsymbol{x}}{\Phi\boldsymbol{R_x}\Phi^{\mathrm{T}}+\sigma}$

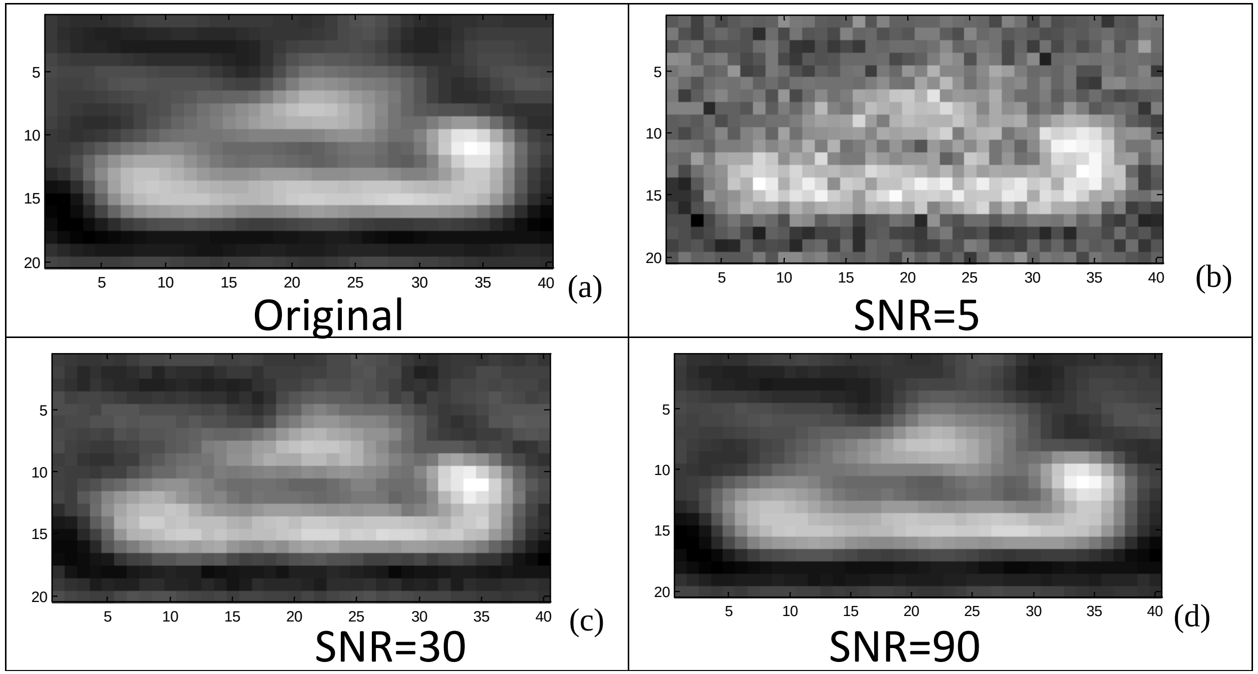
- The residual correlation matrix is
  - $\boldsymbol{R_{\hat{x}}} = \boldsymbol{R_x} + \left(\frac{\boldsymbol{R_x}\Phi^{\mathrm{T}}\Phi\boldsymbol{R_x}}{\Phi\boldsymbol{R_x}\Phi^{\mathrm{T}}+\sigma}\right)\left[\frac{\Phi\boldsymbol{R_x}\Phi^{\mathrm{T}}}{\Phi\boldsymbol{R_x}\Phi^{\mathrm{T}}+\sigma} - 2\right]$

- If σ is small then $\frac{\Phi\boldsymbol{R_x}\Phi^{\mathrm{T}}}{\Phi\boldsymbol{R_x}\Phi^{\mathrm{T}}+\sigma} \cong 1$, then $\boldsymbol{R_{\hat{x}}} = \boldsymbol{R_x} - \lambda\Phi^{\mathrm{T}}\Phi$
  - i.e. this reduces to the conventional eigenvector/eigenvalue problem

Simulations



(I) Typical images used for
estimating target statistics and mask optimization

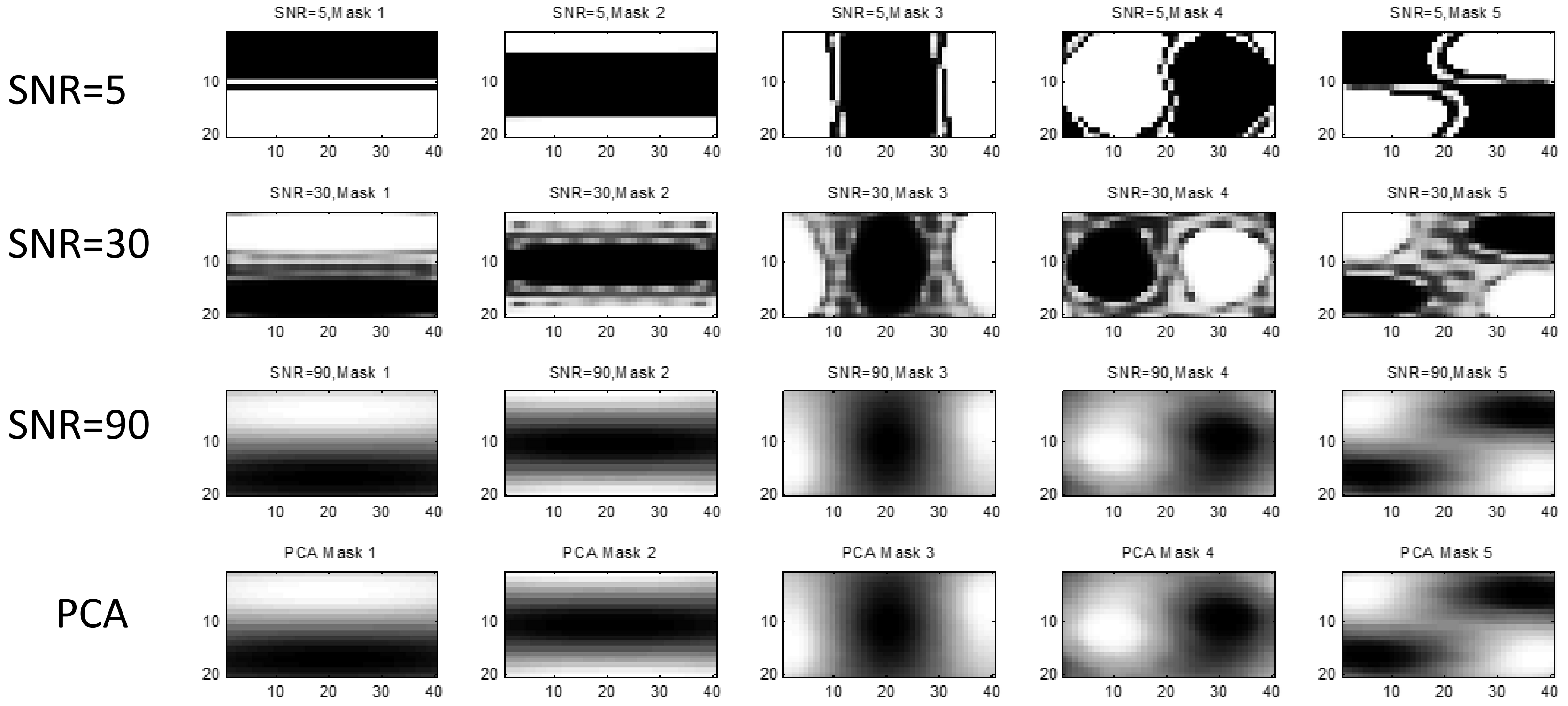


(II)

- Infra red target images are show in (I)
- Conventional images of a target at various noise levels are show in (II)
  - (a) no noise, (b) SNR=5, (c) SNR=30, and (d) SNR=90.

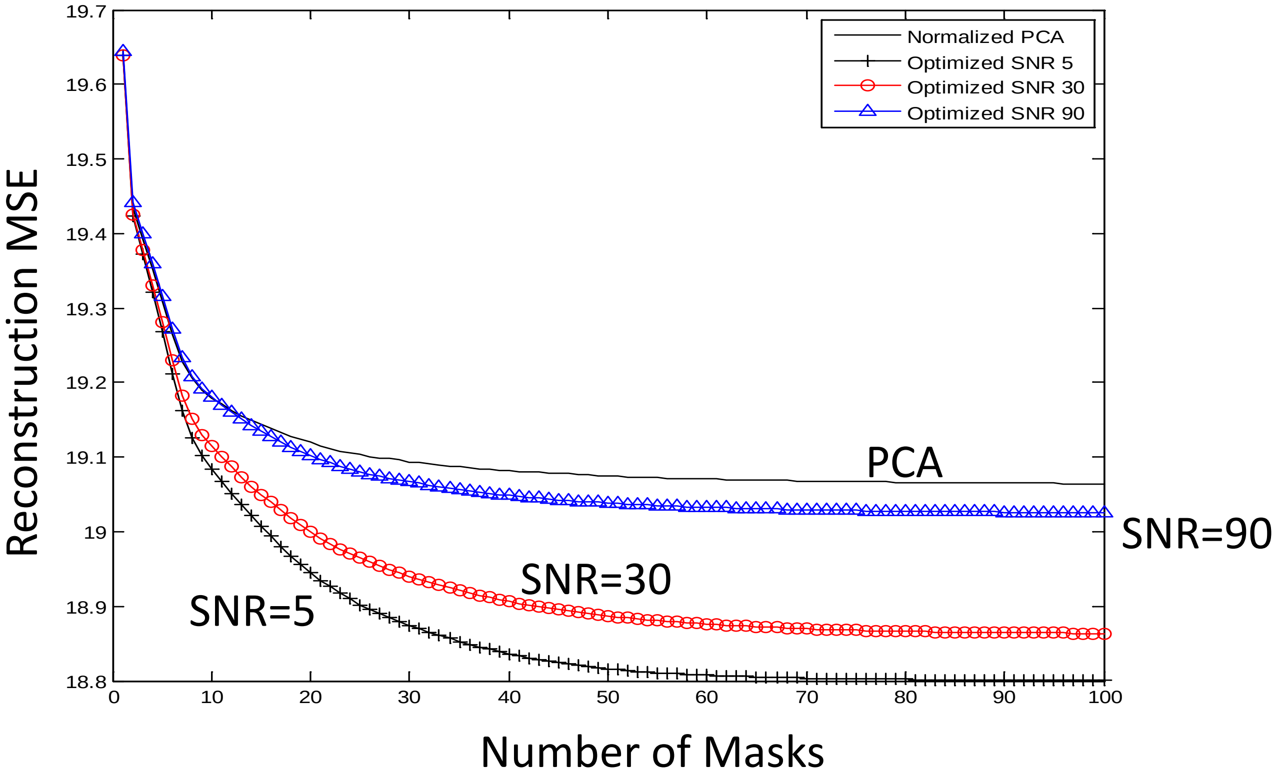
Examples of optimized masks for different Noise Levels

Masks for top 5 optimum measurements



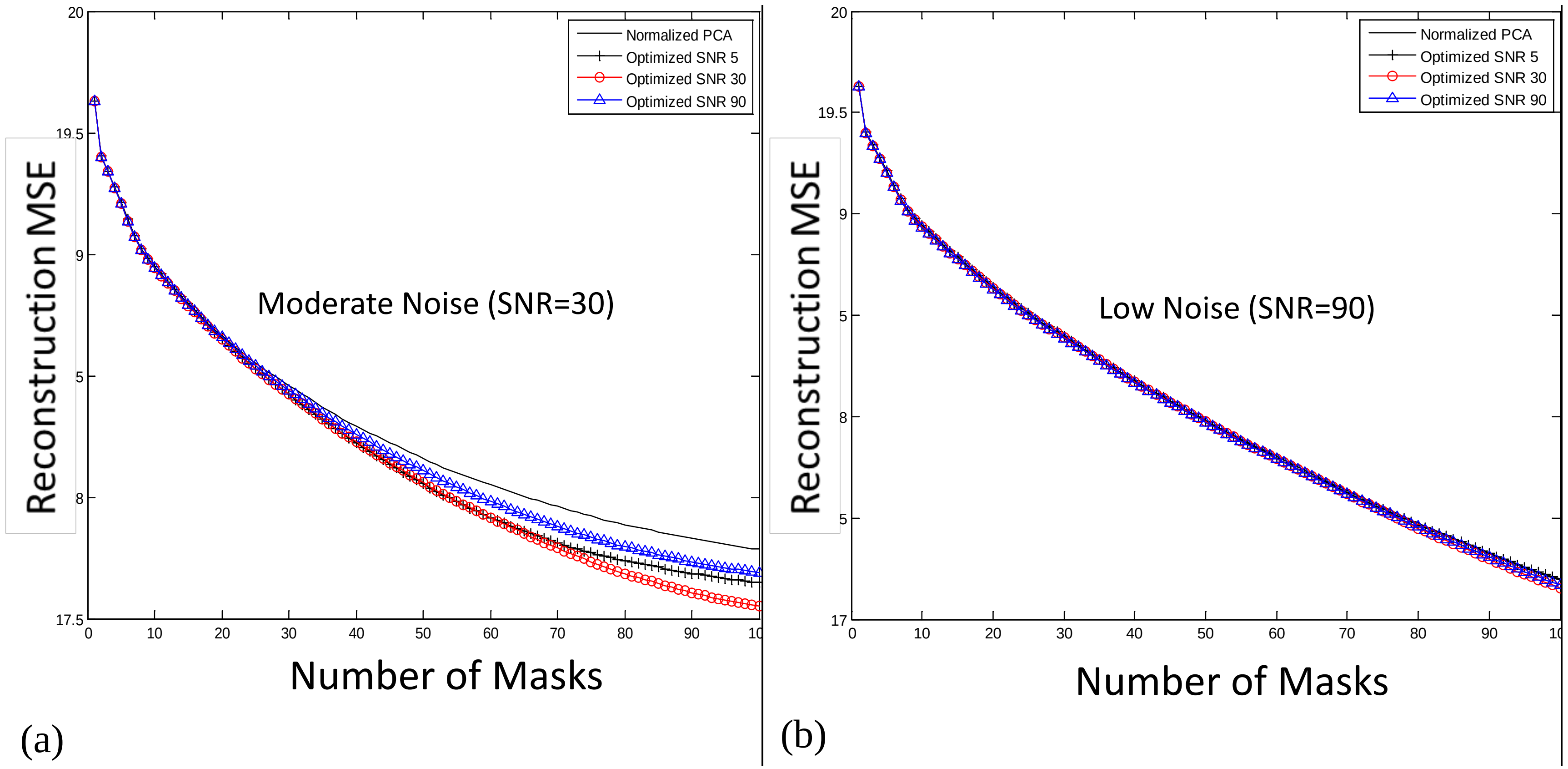
- The masks in the top three rows are optimized for SNR values of 5, 30, and 90 respectively.
- The PCA masks are shown in the bottom row.
  - Note that the masks appear to be almost binary valued for SNR=5 (high noise) and essentially the same as the PCA for SNR=90 (low noise)

Reconstruction error – **high noise** (SNR=5) (without photon constraints)



- In high noise, the masks optimized for an SNR of 5 yield the smallest MSE.
  - Masks optimized for moderate and high SNR also perform better than the PCA.

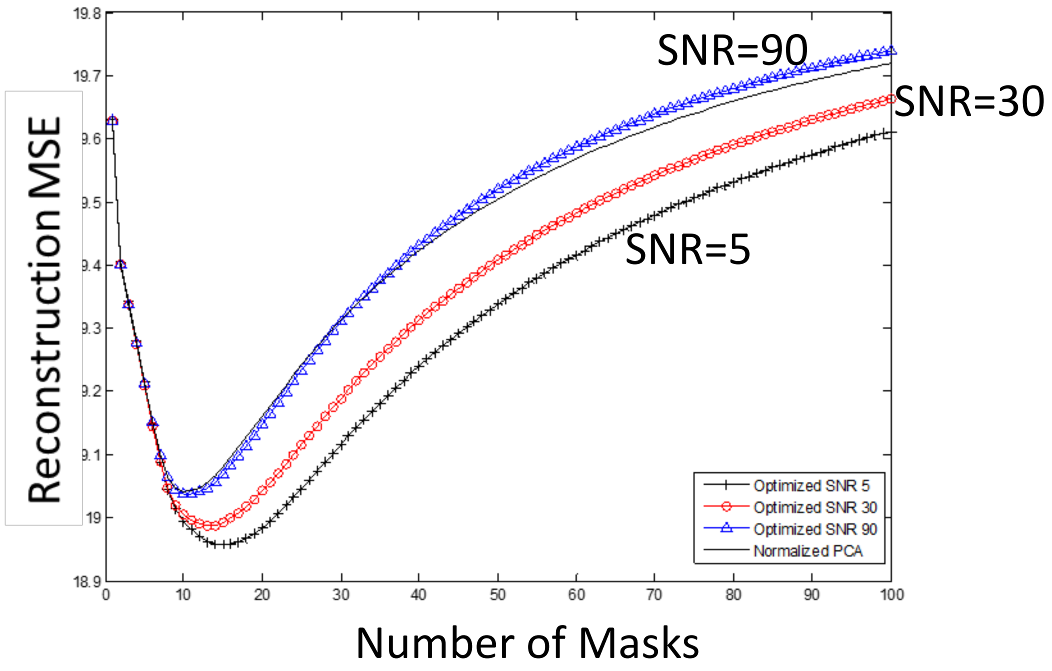
Reconstruction error – **moderate and low noise** (no photon constraint)



- In moderate noise (a), the mask optimized for SNR=30 yields the best performance as the number of features is increased, while the PCA yields highest MSE.

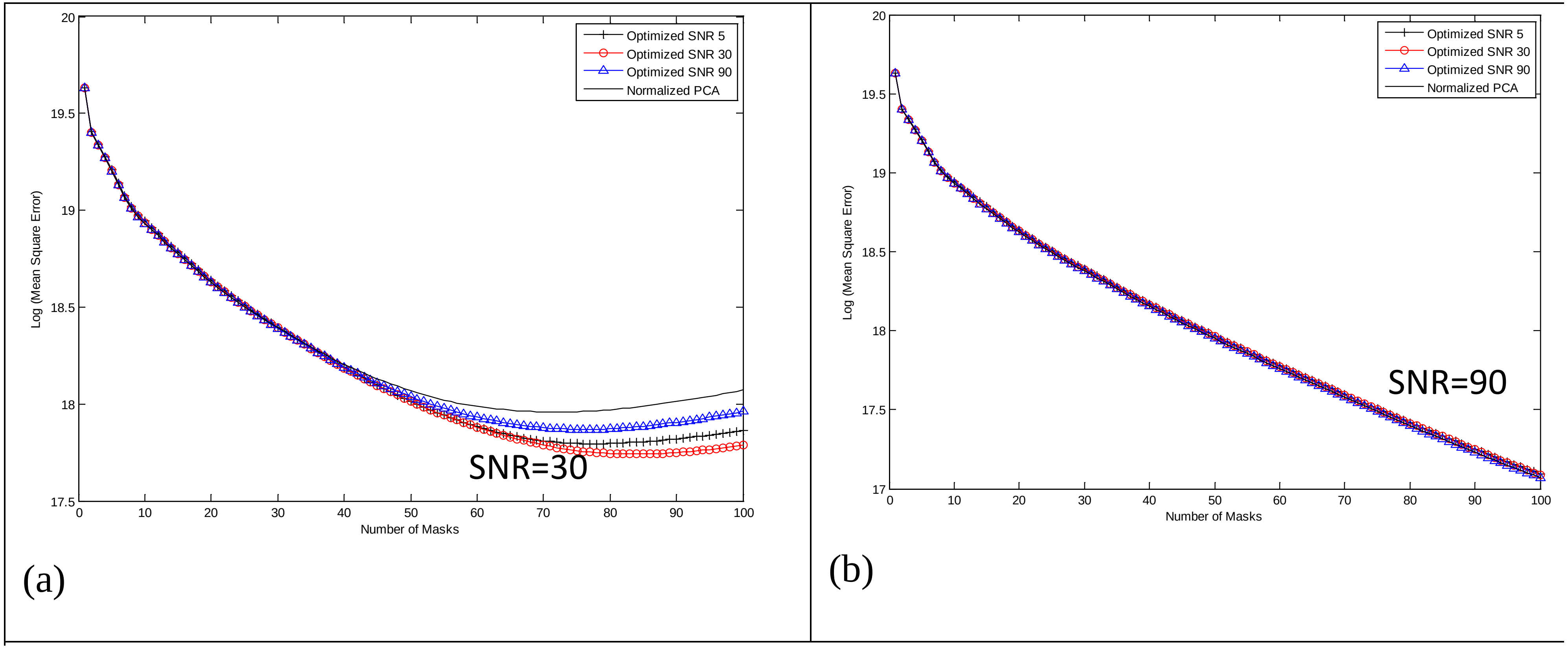
- For SNR = 90 (b), (i.e. in low noise) the performance of all sets of masks is comparable.

Results with Photon Constraints – **high noise**



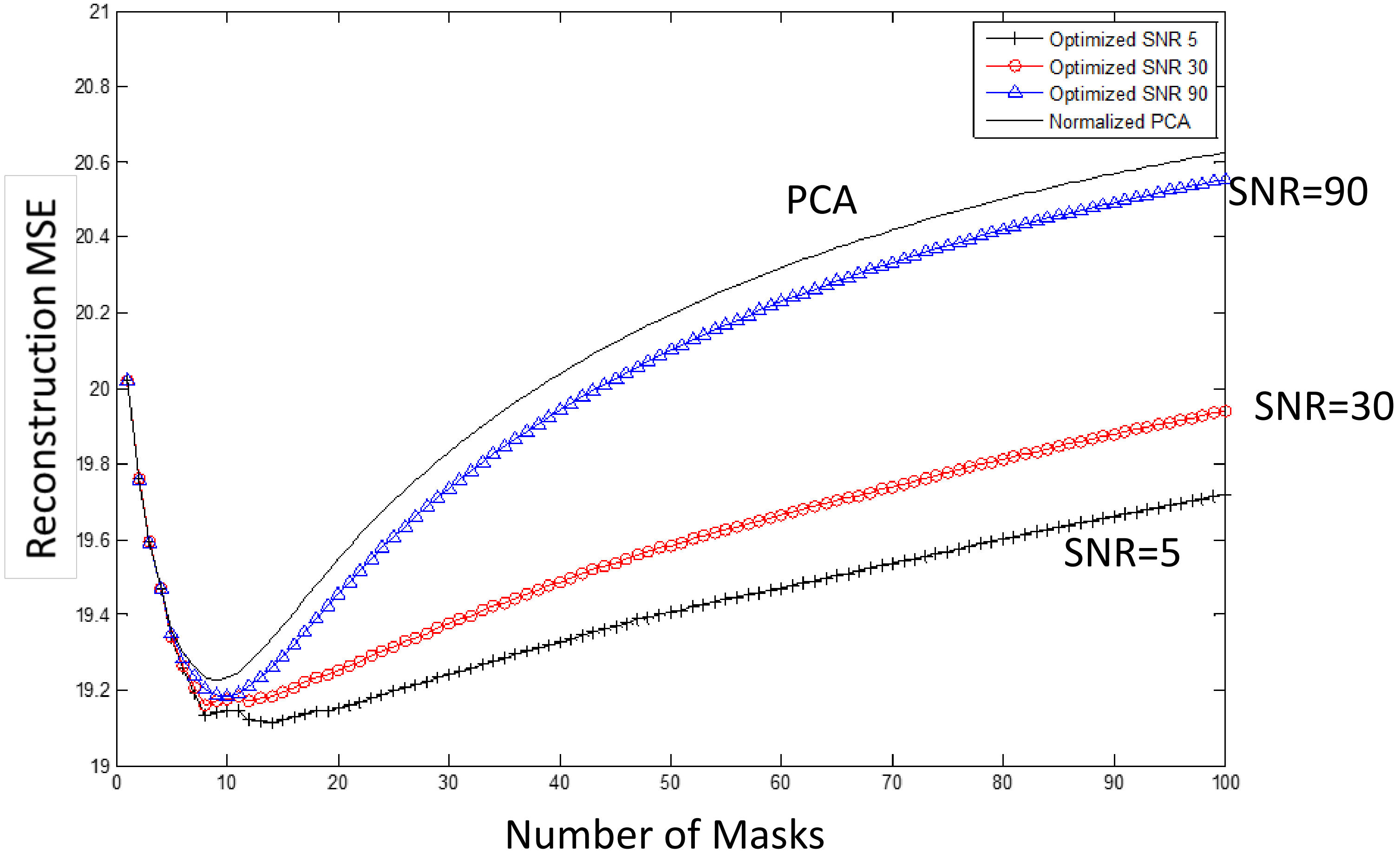
- When a photon constraint is imposed by limiting the integration time allocated to each mask, the MSE initially decreases but then increases again as more noisy measurements are included.
  - In high noise conditions, the best results are obtained using the masks optimized for SNR=5, although the mask optimized for low and medium SNR still outperform the PCA

Results with Photon Constraints – moderate and Low noise



- Under the photon constraint at an SNR =30, the mask optimized for moderate noise yields the best result, compared to the PCA and the masks optimized for other noise levels, as shown in (a).
- In low noise conditions (SNR=90), there is no appreciable difference between any of the masks, as shown in (b).

Signal Dependent Noise



- The masks designed to minimize MSE in **signal independent noise** continue to perform better than the normalized PCA, even when **signal dependent noise** is present.

Examples of Reconstruction at different SNRs

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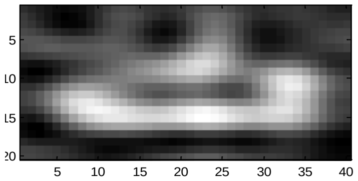
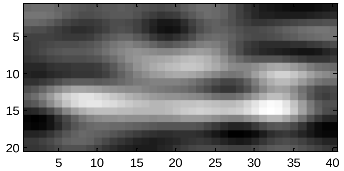
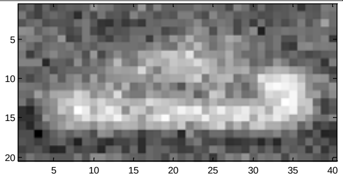
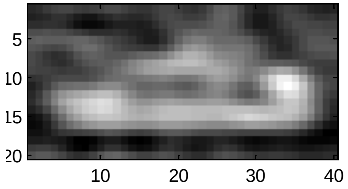
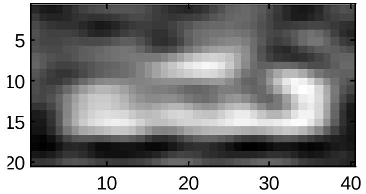
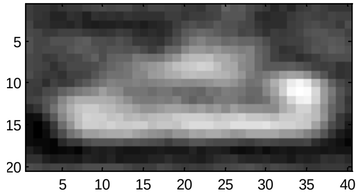
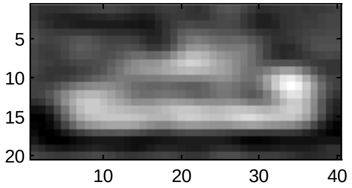
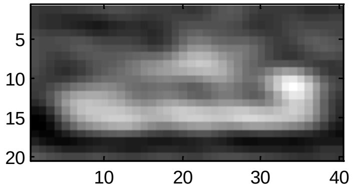
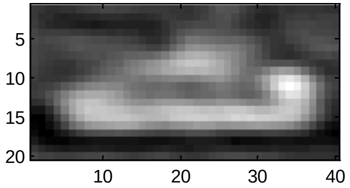
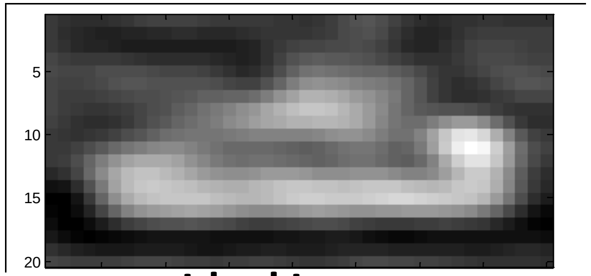
SNR	Optimized measurements	Normalized PCA	Conventional Image
5	 MSE=20.15 Compression Ratio= 47	 MSE=28.8 Compression Ratio =47	 MSE=26.2
30	 MSE=11.1 Compression Ratio = 9.4	 MSE=14.6 Compression Ratio=9.4	 MSE=4.6
90	 MSE=4.61 Compression Ratio=8x	 MSE=4.62 Compression Ratio=8x	 MSE=1.5



Ideal Image

- The results of reconstructing the ideal image using noisy feature specific measurements are shown
  - Compared to PCA and conventional noisy image
- The optimized masks always outperform the PCA by yielding a smaller MSE at the same compression ratio.
- The results are also better than the conventional image in high noise,
  - Visually comparable to the conventional image in moderate and low noise conditions.
  - Reconstruction based on Feature Specific Imaging exhibit a residual MSE due to the compressive nature of the measurements.

Summary

- In EO/IR Compressive Sensing comparatively little attention has been given to the issues that arise when compressive measurements are made in hardware.
  - compressive measurements are corrupted by detector noise.
  - the number of photons available is the same whether a conventional image is sensed, or multiple coded measurements are made in the same interval of time.
  - Thus it is essential that the effects of noise and the constraint on the number photons must be taken into account in the analysis, design and implementation of a compressive imager.
- Feature specific imaging (FSI) is a form of compressive sensing where the measurement kernels are not random, but are based on prior knowledge of the information we are interested in sensing.
  - We have developed a methodology for designing a set of masks that satisfy the photon constraint and are optimum for making measurements that minimize the reconstruction MSE in the presence of noise.
  - To simplify the optimization process, we employed an analytical mapping that ensures the masks can take on any value between ± 1 and formulated a quadratic objective function that can be minimized using gradient descent.
  - The process then finds the mask one at a time, by determining the vector which yields the best possible measurement for reducing the MSE.
  - The sub-space represented by the optimized mask is removed from the signal space, and the process is repeated to find the next best measurement.
- We demonstrated that the photon constraint limits the number of masks that can be used at a particular SNR to reduce the reconstruction MSE.
  - In noisy conditions, MSE initially decreases as the number of measurements is increased, but then increases when measurements that contain more noise than signal information are included.
  - we found that the optimized masks perform better than the normalized PCA, even in signal dependent noise.